Extension of Berenger's PML Absorbing Boundary Conditions to Arbitrary Anisotropic Magnetic Media

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***Abstract*—To simply and effectively absorb waves propagating in arbitrary anisotropic magnetic media, a material independent perfectly matched layer (MIPML) absorber is proposed. Within this MIPML absorber, conductivities σ^E and σ^B (instead of σ^E and σ^H) are used. This results in that electric field E and magnetic flux density B are directly absorbed by the proposed absorber, whereas magnetic field H is simultaneously absorbed through the relation of B and H. It is shown that, with the help of this MIPML absorber, Berenger's PML can be simply and successfully extended to arbitrary anisotropic magnetic media.**

***Index Terms*— Absorbing boundary conditions, anisotropic magnetic media, FDTD.**

I. Introduction

SINCE the perfectly matched layer (PML) absorbing boundary condition was developed by Berenger [1] for absorbing waves propagating in isotropic media, much attention has been paid to its extension to include lossy materials [2], diagonal anisotropic dielectric and/or magnetic materials [3]–[5], and arbitrary anisotropic dielectric media [6]. To further enhance the capability of Berenger's PML technique, extending it to arbitrary anisotropic magnetic materials obviously needs to be considered.

The matching conditions for the most previously reported PML's [1]–[5] are closely related to the permittivity $[\varepsilon]$ and/or permeability $[\mu]$ of the material under consideration. Of course, this does not produce any troubles if the material is isotropic or diagonal anisotropic one, because quite simple matching conditions can easily be derived. It is obvious, however, that if the material is *arbitrary* anisotropic one, the derivation of the PML matching conditions for this material is rather complicated, and it was proven that sometimes no matching conditions can be obtained for arbitrary anisotropic media in three dimensions (3-D) [7]. Very recent investigation [6] indicates that the above difficulties can be easily overcome if the material-independent PML, instead of the traditional PML, is employed. However, the MIPML proposed in [6] was restricted to dielectric materials. In this letter, it will be shown that, by introducing the magnetic flux density $\mathbf{B}$ into the finite-difference time-domain (FDTD) model, Berenger's PML absorbing boundary conditions can be simply and effectively extended to arbitrary anisotropic magnetic media. In particular, in the proposed MIPML absorber the conductivities relating to $\mathbf{E}$ and $\mathbf{B}$ (instead of $\mathbf{E}$ and $\mathbf{H}$) are used and, therefore, a very simple (material independent) matching condition can be derived no matter how complex the permeability tensor $[\mu]$ is. It should be mentioned here that a similar idea for constructing a kind of material-independent PML absorbers was proposed in [8] with an application to lossy (isotropic) materials. Furthermore, the way of implementing the MIPML absorber adopted here is similar to that used in the original Berenger's PML [1], whereas an alternative way was employed in [8]. Validation of the proposed MIPML is carried out through a numerical example.

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II. Theory

Suppose that the relative permeability tensor $[\mu]$ of an arbitrary anisotropic magnetic material in the xy-plane is

$$\mu_{xx} = \mu_1 \cos^2\phi + \mu_2 \sin^2\phi \tag{1.1}$$
$$\mu_{yy} = \mu_1 \sin^2\phi + \mu_2 \cos^2\phi \tag{1.2}$$
$$\mu_{xy} = \mu_{yx} = (\mu_2 - \mu_1)\sin\phi\cos\phi \tag{1.3}$$

where ϕ is the angle between the material axis and the x direction. Hence, waves propagating in such an anisotropic medium within this xy-plane can be categorized into two different waves: TE wave (with E_x, E_y, and H_z field components) and TM wave (with E_z, H_x, and H_y field components). Here only the TM wave will be considered because the TE wave can easily be treated. If a variant of Yee's unit cell (where B_x and B_y locate the same positions as H_x and H_y, respectively) is adopted, then the TM wave in the proposed MIPML absorbers is governed by

$$\frac{\partial E_{zx}}{\partial t} + \frac{\sigma_x^E}{\varepsilon_0}E_{zx} = \frac{1}{\varepsilon_0}\frac{\partial H_y}{\partial x} \tag{2.1}$$
$$\frac{\partial E_{zy}}{\partial t} + \frac{\sigma_y^E}{\varepsilon_0}E_{zy} = -\frac{1}{\varepsilon_0}\frac{\partial H_x}{\partial y} \tag{2.2}$$
$$\frac{\partial B_x}{\partial t} + \sigma_y^B B_x = -\frac{\partial(E_{zx}+E_{zy})}{\partial y} \tag{2.3}$$
$$\frac{\partial B_y}{\partial t} + \sigma_x^B B_y = \frac{\partial(E_{zx}+E_{zy})}{\partial x} \tag{2.4}$$

where the parameters σ_x^E, σ_y^E, σ_x^B, and σ_y^B (which, respectively, relate to $\mathbf{E}$ and $\mathbf{B}$) are the conductivities used in the proposed MIPML absorbers and the matching conditions are

$$\frac{\sigma_x^E}{\varepsilon_0} = \sigma_x^B \quad \text{and} \quad \frac{\sigma_y^E}{\varepsilon_0} = \sigma_y^B. \tag{3}$$

The expression of the theoretical reflection factor $R(\theta)$ for the conductivity $\sigma^E(\rho)$ is exactly the same as that used in [1]. However, because within the proposed MIPML absorbers $\mathbf{B}$ (instead of $\mathbf{H}$) is directly absorbed, updating equations used for $\mathbf{B}$ are slightly different from those used for $\mathbf{H}$ [1]. For example, inside the MIPML absorbers equation (2.4) should read

$$\begin{aligned} B_y^{n+0.5}&(i+0.5,j) \\ &= e^{-\sigma_x^B(i+0.5)\Delta t} B_y^{n-0.5}(i+0.5,j) \\ &\quad + \frac{\left(1-e^{-\sigma_x^B(i+0.5)\Delta t}\right)}{\sigma_x^B(i+0.5)\Delta x}\left[E_{zx}^n(i+1,j)\right. \\ &\quad \left. + E_{zy}^n(i+1,j) - E_{zx}^n(i,j) - E_{zy}^n(i,j)\right]. \end{aligned} \tag{4}$$

From the above discussion one can see that once the conductivity σ^H is replaced by the conductivity σ^B, the proposed MIPML (for both the interfaces and corner regions) can simply be constructed in a similar manner as that used in Berenger's PML [1] for isotropic media. In addition, the relationship between $\mathbf{H}$ and $\mathbf{B}$ in the proposed MIPML absorbers (also inside the anisotropic medium) is

$$H_x = \frac{\mu_{yy}}{\mu_0\left(\mu_{xx}\mu_{yy}-\mu_{xy}^2\right)}B_x - \frac{\mu_{xy}}{\mu_0\left(\mu_{xx}\mu_{yy}-\mu_{xy}^2\right)}B_y \tag{5.1}$$

$$H_y = \frac{\mu_{xx}}{\mu_0\left(\mu_{xx}\mu_{yy}-\mu_{xy}^2\right)}B_y - \frac{\mu_{xy}}{\mu_0\left(\mu_{xx}\mu_{yy}-\mu_{xy}^2\right)}B_x. \tag{5.2}$$

Equations (5.1) and (5.2) indicate that $\mathbf{H}$ can be calculated from $\mathbf{B}$ by using an averaging (with respect to the locations of the field components of $\mathbf{B}$) scheme. This also means that inside the MIPML the H_x and H_y field components can be *concurrently* absorbed while the B_x and B_y field components are *directly* absorbed by the MIPML absorbers.

III. Numerical Validations

To validate the theory developed above, the TM wave propagating in an arbitrary anisotropic magnetic medium (with $\mu_1 = 2.0$ and $\mu_2 = 3.0$) is studied. For all simulations, the following parameters are used: the total computational domain (including the MIPML) is 40 × 40, the space steps are $\Delta x = \Delta y = 6$ mm, the time step is $\Delta t = 12.5$ ps, the number of the MIPML cells (in both the x and y directions) is $N = 10$, a cubic distribution of the conductivity $\sigma^E(\rho)$ with a normal theoretical reflection $R(0) = 0.001\%$ in the MIPML is used, and the results are recorded at 150 time steps. The system is excited by E_z with a smooth compact pulse located at the center of the domain. Especially, the pulse located at the center point (21, 21) of the domain is

$$\begin{aligned} E_z(21,21) = \frac{1}{32}&(10 - 15\cos(2\times 10^9\pi t) \\ &+ 6\cos(4\times 10^9 \pi t) - \cos(6\times 10^9\pi t)) \end{aligned} \tag{6.1}$$

$$E_z(21,21) = 0, \qquad \text{if } t > 80\Delta t. \tag{6.2}$$

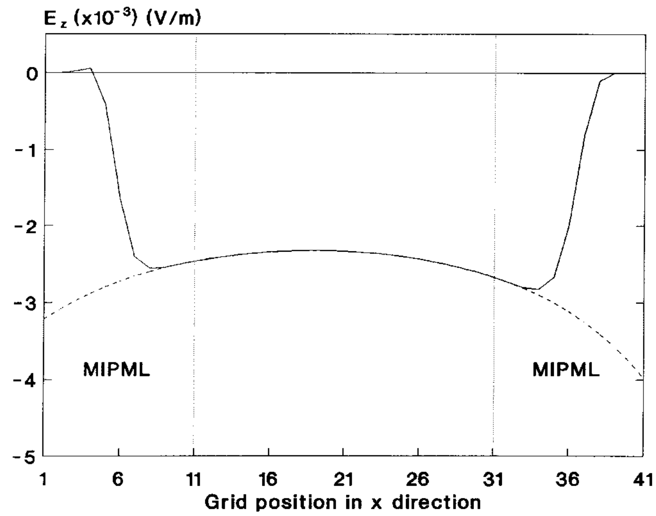

Fig. 1. E_z field profiles of the wave along line $(x, 11)$, where the dashed line is the reference solution and the solid line is the solution obtained with the proposed MIPML.

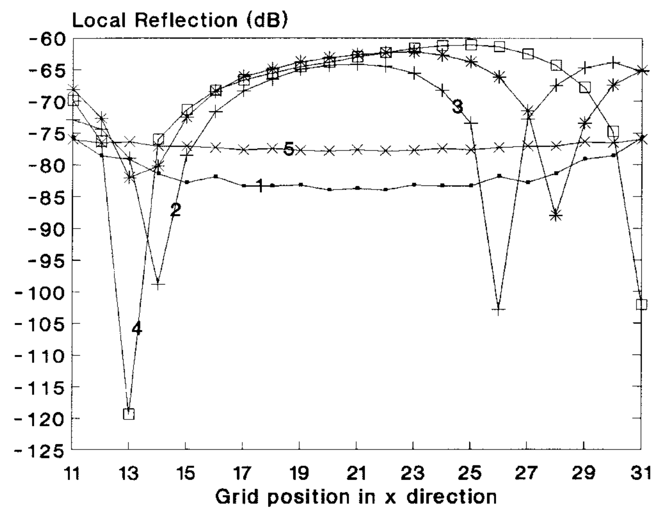

Fig. 2. Normalized local reflections for the E_z field component of the wave along line $(x, 11)$ against ϕ, where curves 1–5 represent $\phi = 0°$, $30°$, $45°$, $60°$, $90°$, respectively.

The reference FDTD solution was computed using a much larger computational domain. To demonstrate how the wave ($\phi = 30°$) is absorbed by the MIPML, the reference solution and the solution obtained with the MIPML for the E_z field component of the wave along line $(x, 11)$ (i.e., the interface between the anisotropic medium and the MIPML) are shown in Fig. 1. It can be seen from Fig. 1 that this wave is effectively absorbed by the proposed MIPML absorbers. To further evaluate the performance of the MIPML absorbers, Fig. 2 shows the normalized (with respect to the peak values of the reference solution along the line $(x, 11)$) local reflections, against ϕ, caused by the proposed MIPML for the E_z field component of the wave along the interface. From Fig. 2, one can see that the proposed MIPML performs quite well (better than 60 dB) for all these different anisotropic cases. Besides, it is not surprising that due to the anisotropy, different absorbing performances are observed since in the construction of the MIPML absorber same parameters were used for different values of ϕ. In addition, it is anticipated that a better absorbing performance can be obtained if an optimization procedure on selection of the parameter of the MIPML absorbers is adopted.

IV. Conclusion

To extend Berenger's PML to arbitrary anisotropic magnetic media, a material-independent PML absorber is proposed by introducing the $\mathbf{B}$ field into the FDTD model. Due to the fact that inside the proposed MIPML absorbers the conductivities σ^E and σ^B (instead of σ^E and σ^H) are used, the absorbers developed in this letter have the property of material independence. Extension of the proposed MIPML to 3-D cases is straightforward. Furthermore, due to the special feature (i.e., material independence) of the proposed MIPML absorbers, it can also be applied to structures consisting of arbitrary anisotropic magnetized ferrites, nonlinear, and dispersive materials with slight modifications.

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